Principal Equations of Dialysis

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An Equation is...

• Math:
  • “A statement that each of two statements are equal to each other.”
    \[ Y^2 = 3x^3 + 2x + 7 \]

• Chemistry:
  • “A symbolic expression that represents a chemical change as observed in a laboratory”.
    \[ 2H_2O = H_3O^+ + OH^- \]

• Medical:
  • “An expression made up of two members connected by the sign of equality”.
    \[ \text{Clearance x Time} = \text{Volume} \ (Kt/V = 1) \]
Equation Types

• Hypothesis
  • Relationships implied without supporting evidence

• Empirical
  • Based solely on experiment and observation
  • No reference to scientific principals

• Theoretical
  • A formulation of apparent relationships
  • Deals with science concepts and knowledge
  • Implies considerable evidence of support
  • Pure science as opposed to applied science
# K/DOQI Guidelines for Classification

<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
<th>GFR (mL/min)</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Damage with normal or high GFR</td>
<td>&gt;90</td>
<td>CVD risk reduction; diagnose and treat; slow progression</td>
</tr>
<tr>
<td>2</td>
<td>Mild decrease in GFR</td>
<td>60-89</td>
<td>Monitor progression; nutritional assessment and intervention</td>
</tr>
<tr>
<td>3</td>
<td>Moderate decrease in GFR</td>
<td>30-59</td>
<td>Evaluate and treat complications</td>
</tr>
<tr>
<td>4</td>
<td>Severe decrease in GFR</td>
<td>15-29</td>
<td>Prepare for replacement therapy</td>
</tr>
<tr>
<td>5</td>
<td>Kidney Failure</td>
<td>&lt;15</td>
<td>Replacement therapy if uremia is present</td>
</tr>
</tbody>
</table>
Question:

Is there a way to determine which classification a patient falls into and what information do we need to know to figure this out?
MDRD Study Equation for calculating GFR

GFR (mL/min per 1.73 m² body surface area) =

\[186 \times (S_{\text{Cr}})^{-1.154} \times (\text{Age})^{-0.203} \times (0.742 \text{ if female}) \times (1.210 \text{ if African-American})\]

\[S_{\text{Cr}} = \text{serum creatinine measured in mg/dL}\]

- Not validated in:
  - Diabetic kidney disease
  - Patients with serious comorbid conditions
  - Normal persons
  - Persons older than 70.

MDRD = Modification of Diet in Renal Disease
GFR vs. Age and Serum Creatinine
GFR (mg/min) vs. Serum Creatinine (mg/dL) (Gender and Race)
Question:

If we are going to treat the patient, we need a way to measure our success. Urea is the major marker used.

*Is there a way to know how much urea a patient will generate based on their diet intake of protein?*
The Conversion Equation
Protein to Urea Nitrogen

\[ DPI = PCR = 9.35 \, G + 11.04 \]

or

\[ G = \frac{PCR - 11.04}{9.35} \]

DPI = Dietary Protein Intake (grams/day)

PCR = Protein Catabolic Rate (grams/day)

G = Generation Rate (milligrams of urea nitrogen/minute)
BUN Generation Rate vs. Protein Catabolic Rate

BUN Generation Rate (mg/minute) vs. Protein Catabolic Rate (g/kg/day)

Graph showing a linear relationship between BUN Generation Rate and Protein Catabolic Rate.
<table>
<thead>
<tr>
<th>PCR \ g/kg/day</th>
<th>50.0</th>
<th>60.0</th>
<th>70.0</th>
<th>80.0</th>
<th>90.0</th>
<th>100.0</th>
<th>110.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>3.10</td>
<td>3.95</td>
<td>4.81</td>
<td>5.66</td>
<td>6.52</td>
<td>7.38</td>
<td>8.23</td>
</tr>
<tr>
<td>0.90</td>
<td>3.63</td>
<td>4.59</td>
<td>5.56</td>
<td>6.52</td>
<td>7.48</td>
<td>8.44</td>
<td>9.41</td>
</tr>
<tr>
<td>1.00</td>
<td>4.17</td>
<td>5.24</td>
<td>6.31</td>
<td>7.38</td>
<td>8.44</td>
<td>9.51</td>
<td>10.58</td>
</tr>
<tr>
<td>1.10</td>
<td>4.70</td>
<td>5.88</td>
<td>7.05</td>
<td>8.23</td>
<td>9.41</td>
<td>10.58</td>
<td>11.76</td>
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<tr>
<td>1.20</td>
<td>5.24</td>
<td>6.52</td>
<td>7.80</td>
<td>9.09</td>
<td>10.37</td>
<td>11.65</td>
<td>12.94</td>
</tr>
<tr>
<td>1.30</td>
<td>5.77</td>
<td>7.16</td>
<td>8.55</td>
<td>9.94</td>
<td>11.33</td>
<td>12.72</td>
<td>14.11</td>
</tr>
<tr>
<td>1.40</td>
<td>6.31</td>
<td>7.80</td>
<td>9.30</td>
<td>10.80</td>
<td>12.30</td>
<td>13.79</td>
<td>15.29</td>
</tr>
</tbody>
</table>
BUN Increase/Day in a ESRD Patient - Example

• Assume:
  • Patient’s Weight = 70.0 kilograms
  • PCR = 1.2 g/kg/day = Generation rate of 7.80 mg/min
  • Patient’s Fluid Volume = 58% of Patient Weight

• Then:
  • 70 x 0.58 = 40.6 liters = 406 deciliters
  • BUN generated /day = 7.80 mg/min x 1440 min/day = 11,232 mg/day
  • 11,232 mg/day / 406 dL = 27.7 mg/dL/day.
### BUN Generated/day (grams)

<table>
<thead>
<tr>
<th>PCR g/kg/day</th>
<th>50.0</th>
<th>60.0</th>
<th>70.0</th>
<th>80.0</th>
<th>90.0</th>
<th>100.0</th>
<th>110.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>4.46</td>
<td>5.69</td>
<td>6.92</td>
<td>8.16</td>
<td>9.39</td>
<td>10.62</td>
<td>11.85</td>
</tr>
<tr>
<td>0.90</td>
<td>5.23</td>
<td>6.62</td>
<td>8.00</td>
<td>9.39</td>
<td>10.77</td>
<td>12.16</td>
<td>13.55</td>
</tr>
<tr>
<td>1.00</td>
<td>6.00</td>
<td>7.54</td>
<td>9.08</td>
<td>10.62</td>
<td>12.16</td>
<td>13.70</td>
<td>15.24</td>
</tr>
<tr>
<td>1.10</td>
<td>6.77</td>
<td>8.46</td>
<td>10.16</td>
<td>11.85</td>
<td>13.55</td>
<td>15.24</td>
<td>16.94</td>
</tr>
<tr>
<td>1.20</td>
<td>7.54</td>
<td>9.39</td>
<td>11.24</td>
<td>13.08</td>
<td>14.93</td>
<td>16.78</td>
<td>18.63</td>
</tr>
<tr>
<td>1.30</td>
<td>8.31</td>
<td>10.31</td>
<td>12.31</td>
<td>14.32</td>
<td>16.32</td>
<td>18.32</td>
<td>20.32</td>
</tr>
<tr>
<td>1.40</td>
<td>9.08</td>
<td>11.24</td>
<td>13.39</td>
<td>15.55</td>
<td>17.71</td>
<td>19.86</td>
<td>22.02</td>
</tr>
</tbody>
</table>
## Patient BUN Gain/Day (mg/dL)

<table>
<thead>
<tr>
<th>PCR g/kg/day</th>
<th>Patient Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50.0</td>
</tr>
<tr>
<td>0.80</td>
<td>15.38</td>
</tr>
<tr>
<td>0.90</td>
<td>18.04</td>
</tr>
<tr>
<td>1.00</td>
<td>20.69</td>
</tr>
<tr>
<td>1.10</td>
<td>23.35</td>
</tr>
<tr>
<td>1.20</td>
<td>26.00</td>
</tr>
<tr>
<td>1.30</td>
<td>28.66</td>
</tr>
<tr>
<td>1.40</td>
<td>31.31</td>
</tr>
</tbody>
</table>
Patient's PCR vs. Pre Treatment BUN mg/dL
(Patient’s previous post Tx BUN = 25 mg/dL)
Question:

Since the patient will need to have his/her urea removed, and there are so many different dialyzers, is there a simple way to measure urea removal performance for a given dialyzer?
Dialyzer BUN Clearance

THE EMPIRICAL FORMULA FOR BLOOD CLEARANCE IS:

\[
C_X = \left( \frac{A_X - V_X}{A_X} \right) Q_B
\]

WHERE:

\( C_X \) = CLEARANCE OF SOLUTE X. (mL/min)

\( A_X \) = ARTERIAL CONCENTRATION OF X. (mg/dL)

\( V_X \) = VENOUS CONCENTRATION OF X. (mg/dL)

\( Q_B \) = BLOOD FLOWRATE (mL/min)
Urea Clearance in Blood and Dialysate

\[ C_X = \frac{(80 - 10)}{80} \times 300 = 262 \text{ mL/min} \]

**Dialyzer: iP4U**

- \[ K_{OA} = 900 \text{ mL/min} \]
- \[ Q_B = 300 \text{ mL/min} \]
- \[ Q_D = 600 \text{ mL/min} \]

**Equation:**

\[ Q_B \times \Delta \text{BUN}_B = Q_D \times \Delta \text{BUN}_D \]

- \[ 300 \times (80 - 10) = 600 \times (35 - 0) \]
- \[ 300 \times 70 = 600 \times 35 \]
- \[ 21,000 = 21,000 \]
Question:

It’s not very practical to measure blood urea nitrogen concentrations to determine clearance.

*Is there an equation that can calculate the expected clearance based on a known blood flowrate, dialysate flowrate and dialyzer used?*
Determining the $K_oA$ for a Dialyzer

$$K_oA = \left[ \frac{Q_B}{1 - \frac{Q_B}{Q_D}} \right] \ln \left[ 1 - \frac{C_x}{Q_D} \right]$$

Where:
- $C_x$ = Clearance of solute, $X$
- $Q_B$ = Blood flowrate
- $Q_D$ = Dialysate flowrate
- $\ln$ = Natural logarithm
  - $e = 2.718281828\ldots$
Calculating Clearance using KoA

\[ C_x = \frac{Q_B \left( KoA \left( \frac{1}{Q_B} - \frac{1}{Q_D} \right) - 1 \right)}{e \left( KoA \left( \frac{1}{Q_B} - \frac{1}{Q_D} \right) - \frac{Q_B}{Q_D} \right)} \]

\[ C_x = \text{Clearance of } x. \]
\[ Q_B = \text{Blood Flowrate} \]
\[ Q_D = \text{Dialysate Flowrate} \]
\[ Q_B \neq Q_D \]
\[ KoA = \text{Clearance Coefficient} \]
\[ e = 2.718281828\ldots \]
BUN of **Venous Blood** based on Dialyzer **KoA**

<table>
<thead>
<tr>
<th>KoA  (mL/min)</th>
<th>Clearance (mL/min)</th>
<th>QB = 300 mL/min</th>
<th>QD = 600 mL/min</th>
<th>Arterial Blood BUN Values (mg/dL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>217</td>
<td>19.4</td>
<td>22.1</td>
<td>24.9</td>
</tr>
<tr>
<td>600</td>
<td>232</td>
<td>15.9</td>
<td>18.1</td>
<td>20.4</td>
</tr>
<tr>
<td>700</td>
<td>245</td>
<td>12.8</td>
<td>14.7</td>
<td>16.5</td>
</tr>
<tr>
<td>800</td>
<td>254</td>
<td>10.7</td>
<td>12.3</td>
<td>13.8</td>
</tr>
<tr>
<td>900</td>
<td>262</td>
<td>8.9</td>
<td>10.1</td>
<td>11.4</td>
</tr>
<tr>
<td>1000</td>
<td>269</td>
<td>7.2</td>
<td>8.3</td>
<td>9.3</td>
</tr>
<tr>
<td>1100</td>
<td>274</td>
<td>6.1</td>
<td>6.9</td>
<td>7.8</td>
</tr>
<tr>
<td>1200</td>
<td>278</td>
<td>5.1</td>
<td>5.9</td>
<td>6.6</td>
</tr>
</tbody>
</table>
Clearance vs. KoA
(Qd = 600 mL/min)
Question:

Once the fluid volume of the patient is known and the dialyzer clearance calculated, is there an equation to determine the time of dialysis?
Length of Treatment

\[ \text{Kt/V} = 1.3 \]

\[ K = \text{Dialyzer Clearance (mL/min)} \]
\[ t = \text{Time of Treatment (min)} \]
\[ V = \text{Patient’s volume (mL)} \]

\[ t = \frac{(1.3 \times V)}{K} \]
Treatment Time (minutes) vs. Kt/V
(Patient = 70 kg, $C_x = 262 \text{ mL/min}$)
Question:

*Once the clearance of the dialyzer and time of treatment are known, is there a way to estimate how the urea is reduced in the patient while she/he is being dialyzed?*
Urea Reduction Equation

\[ C = C_0 e^{-Kt/V} + \frac{G}{K} (1 - e^{-Kt/V}) \]

- \( C \) = Plasma BUN Concentration (mg/mL)*
- \( C_0 \) = Predialysis BUN Concentration (mg/mL)*
- \( K \) = Dialyzer Clearance (mL/min)
- \( t \) = time (minutes)
  - \( V \) = Patient Volume (mL)
- \( G \) = Generation of urea (mg/min)

*mg/mL equals mg/dL divided by 100.
Patient's Mid-week Urea Reduction

BUN (mg/dL) vs. TIME (minutes)
Question:

If the urea concentration at the beginning and end of the treatment are known, is there a relationship between Kt/V and these values?
**URR and Kt/V**

- **URR = Urea Reduction Ratio**
  \[
  URR = \frac{C_{\text{PRE}} - C_{\text{POST}}}{C_{\text{PRE}}}
  \]
  Patient = \frac{(80 - 24)}{80} = 0.70

- **Kt/V = Dialysis Treatment Index**
  \[
  Kt/V = \text{Clearance} \times T_x \times \text{time} \div \text{Patient V}
  \]
  Patient (in vitro) = \frac{262 \times 240}{40,600} = 1.55
  Patient (in vivo) = 80\% \text{ of In Vitro} = 1.24


Patient’s Kt/V

\[ C_{\text{PRE}} = 80 \text{ mg/dL} \quad C_{\text{POST}} = 24 \text{ mg/dL} \quad URR = 70\% \]

\[ R = \frac{C_{\text{POST}}}{C_{\text{PRE}}} \quad \text{UF} = \text{Fluid Removed} \quad W = \text{Post Weight} \]

\[ R = \frac{24}{80} = 0.30 \quad \text{UF} = 0.0 \text{ kg} \quad W = 70 \text{ kg} \]

\[
\begin{align*}
\text{Kt/V} &= 2.2 - 3.3(R - 0.03 - \frac{\text{UF}}{W}) \\
&= 2.2 - 3.3[0.30 - 0.03 - (0/70)] \\
&= 2.2 - 3.3(0.30 - 0.03 - 0.0) \\
&= 2.2 - 3.3(0.27) = 2.2 - 0.89 \\
&= 1.31
\end{align*}
\]
**UF Effect on Kt/V**

Kt/V = 2.2 – 3.3(R – 0.03 – UF/W)

R = \( C_{\text{POST}} / C_{\text{PRE}} \)  \( \text{UF} = \text{Fluid Removed} \)  \( \text{W} = \text{Post Weight} = 70 \text{ kg} \)

Column 3 = Post BUN effective reduction due to UF (mg/dL)

Column 4 = Needed dialyzer clearance for reduction in column 3

<table>
<thead>
<tr>
<th>( T \times \text{UF (kg)} )</th>
<th>( R = 0.30 )</th>
<th>Column 3</th>
<th>Column 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.31</td>
<td>24</td>
<td>221</td>
</tr>
<tr>
<td>1.0</td>
<td>1.36</td>
<td>22.8</td>
<td>230</td>
</tr>
<tr>
<td>2.0</td>
<td>1.40</td>
<td>21.8</td>
<td>236</td>
</tr>
<tr>
<td>3.0</td>
<td>1.45</td>
<td>20.6</td>
<td>245</td>
</tr>
<tr>
<td>4.0</td>
<td>1.50</td>
<td>19.4</td>
<td>253</td>
</tr>
<tr>
<td>5.0</td>
<td>1.54</td>
<td>18.4</td>
<td>260</td>
</tr>
</tbody>
</table>

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Achieving Quality Therapy

1. **Prescribe Desired Kt/V**
2. **Measure Treatment URR**
3. **Calculate Treatment Parameters K, t, and V**
4. **Compare URR vs. Kt/V**

The cycle is completed by **Delivering** the therapy parameters and **Adjusting** as necessary.
References


• **NKF Practice Guidelines for Chronic Kidney Disease: Evaluation, Classification, and Stratification** – Andrew S. Levey, MD, et al, Annals of Internal Medicine, Volume 139, No. 2, July 2003
